CALIBRATION ALGORITHM FOR OPTIMAL ANGLE CALCULATION

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INTRODUCTION

This document describes how to implement different possible calibration algorithms to achieve optimal accuracy in angle calculation for Allegro angle sensors with analog sine-cosine outputs.

The described procedure applies to the Allegro A33230 3D Hall-effect sensor IC with sine/cosine outputs.

CALIBRATION PARAMETERS

Angle sensors with analog output provide one couple of sinusoidal analog signals x, y that can be expressed by Equation 1.

Equation 1:

$$x = A_X \times \cos(\theta + \varphi_X) + O_X$$

$$y = A_Y \times \sin(\theta + \varphi_Y) + O_Y$$

where θ is the angle, A_X, A_Y are the amplitudes of cosine and sine signals, O_X and O_Y are the offset values, and φ_X and φ_Y are the phase with respect to an arbitrary angle reference.

The phase shift error $\Delta\varphi$ between sine and cosine signal is defined in Equation 2.

Equation 2:

$$\Delta\varphi = \varphi_Y - \varphi_X$$

For an ideal sensor, sine and cosine signals are perfectly orthogonal and $\Delta\varphi = 0$.

A_X, A_Y, O_X, O_Y, and $\Delta\varphi$ are the five calibration parameters needed to compensate signal non-idealities prior to angle calculation. Figure 1 illustrates an example of two output signals and their parameters.

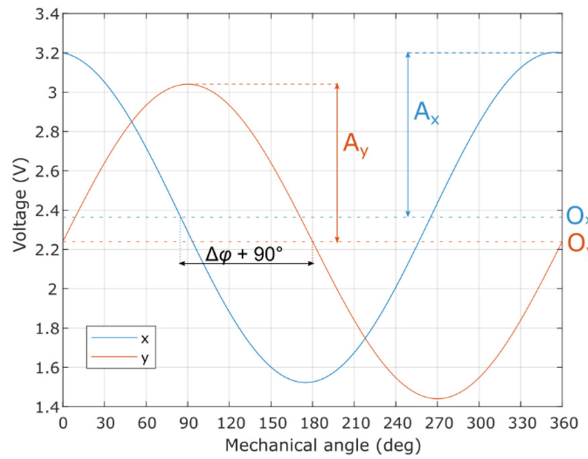

Figure 1: Illustrative of output signals with graphical identification of calibration parameters. Errors are exaggerated for illustrative purposes.

Refer to the part-specific datasheet to find the ranges of the parameters value.

The exact parameters values can be learned during end-of-line static calibration or during an online continuous dynamic calibration as described in the paragraphs below.

HOW TO APPLY CALIBRATION

Once the parameters are known, calibration can be applied to acquired signals **x** and **y** with the following algorithm:

1. Offset removal

$$x_1 = x - O_X$$
$$y_1 = y - O_Y$$

2. Amplitude normalization

$$x_2 = x_1 \; / \; A_X$$
$$y_2 = y_1 \; / \; A_Y$$

3. Orthogonality error compensation

$$x_3 = (\sin(\Delta\varphi) \times y_2 + x_2) \; / \; \cos(\Delta\varphi)$$
$$y_3 = y_2$$

Angle can then be calculate on obtained signals:

$$\theta = \mathrm{atan2}(y_3, x_3)$$

where atan2 is the four-quadrant inverse tangent function.

An example of signals before and after compensation with the relative angular error is provided in Figure 2.

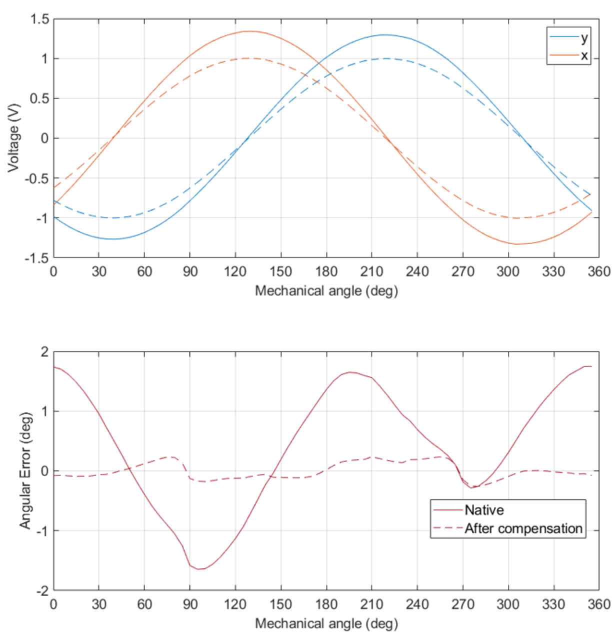

Figure 2: Top: Example of real sensor's signals prior of compensation (continuous line) and after offset, amplitude, and orthogonality correction. Bottom: Error for the angle calculation with native signals and after compensation. Nominal central voltage output was removed in native signals.

END-OF LINE CALIBRATION

This section describes how to learn calibration parameters during an end-of line (EOL) calibration.

Multiple end-of-line calibration algorithms exist. In this section, two different methods both requiring accurate angular position are described.

Note: In order to avoid errors in parameters calculation, all calibration measurements should be taken at the same temperature. Different calibrations might be performed at multiple temperatures to compensate for temperature effects.

Non-Linear Least Square Fitting Method

Non-linear least square (NLLSQR) fitting method requires an accurate reference system. Thanks to its robustness to noise in measurement samples, NLLSQR offers high accuracy in determining calibration parameters with a limited number of sample points that don't need to be equally spaced.

To perform calibration, **x** and **y** outputs should be measured at **N** angle points θ along one rotation. Three vectors of values are thus obtained:

$$\theta = [\theta_1 \;\; \theta_2 \ldots \theta_N]$$
$$x = [x_1 \;\; x_2 \ldots x_N]$$
$$y = [y1 \;\; y_2 \ldots y_N]$$

The sets of values can be fitted with the NLLSQR algorithm to Equations 1 in order to obtain calibration parameters.

Discrete Fourier Transform (DFT) Method

DFT is a powerful calibration method that allows the calculation of not only the basic calibration parameters but also higher harmonic coefficients for more complex higher harmonics compensation.

For the algorithm to work, DFT requires an accurate reference system and N measurement points of **x** and **y** along one full 360° rotation, spaced by the same angle step $\Delta\theta = 360° \; / \; N$.

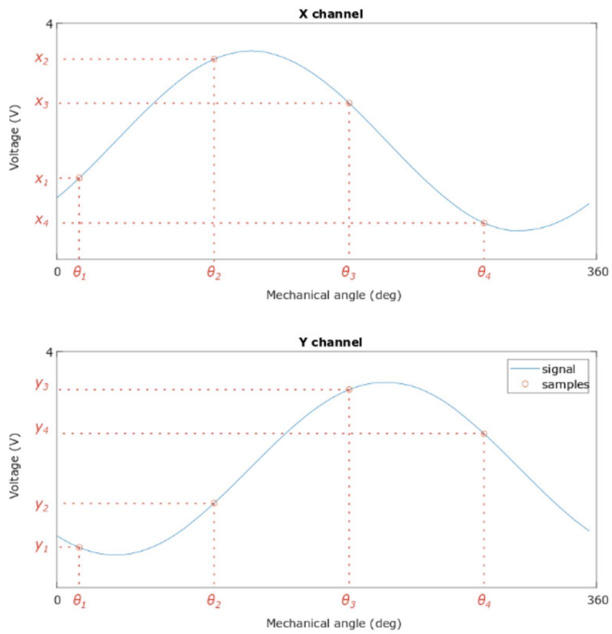

Figure 3: Example of signals and sampling points with N equal to four.

Given at least four measurements points equally spaced every 90° as shown in Figure 3, it is possible to calculate the offset parameters:

$$O_X = (x_1 + x_2 + \cdots + x_N) / N$$
$$O_Y = (y_1 + y_2 + \cdots + y_N) / N$$

To determine amplitude and phase shift parameters, the first term of the DFT real and imaginary part should be calculated for x and y.

$$X_{DFT}\,Re = (x_1 \times \cos(\theta_1) + x_2 \times \cos(\theta_2) + \cdots + x_N \times \cos(\theta_N)) \times 2 / N$$
$$X_{DFT}\,Im = (x_1 \times \sin(\theta_1) + x_2 \times \sin(\theta_2) + \cdots + x_N \times \sin(\theta_N)) \times 2 / N$$
$$Y_{DFT}\,Re = (y_1 \times \cos(\theta_1) + y_2 \times \cos(\theta_2) + \cdots + y_N \times \cos(\theta_N)) \times 2 / N$$
$$Y_{DFT}\,Im = (y_1 \times \sin(\theta_1) + y_2 \times \sin(\theta_2) + \cdots + y_N \times \sin(\theta_N)) \times 2 / N$$

Amplitude and phase shift calibration parameter can then be calculated as:

$$A_X = \sqrt{(X_{DFT}\,Re^2 + X_{DFT}\,Im^2)}$$
$$A_Y = \sqrt{(Y_{DFT}\,Re^2 + Y_{DFT}\,Im^2)}$$
$$\Delta\varphi = \mathrm{atan2}(Y_{DFT}\,Im, Y_{DFT}\,Re) - \mathrm{atan2}(X_{DFT}\,Im, X_{DFT}\,Re)$$

Online Continuous Calibration

This section describes how to continuously learn calibration parameters during device operation.

With a continuous calibration, it is possible to compensate for effects due to temperature and aging of the system that cannot be predicted with end-of-line calibration methods.

With respect to end-of-line calibration, it is not possible to have an accurate reference angular measurement beyond the one provided by the sensor itself, the proposed methods will thus have to use only the sensor outputs.

Note: For correct calibration, it is essential that the measured samples used for parameters calculation are taken at the same temperature. A full online calibration loop should happen in a reasonably short amount of time to avoid temperature drifts between the needed sampling events.

Offset and Amplitude Parameters

Online continuous calibration is based on the maximum and minimum tracking on the output signals x and y over a full 360° rotation. The parameters indicated should be thus known: X_{max}, X_{min}, Y_{max}, Y_{min}. Offset and amplitude parameters can then be calculated as:

$$O_X = (X_{max} + X_{min}) / 2$$
$$O_Y = (Y_{max} + Y_{min}) / 2$$
$$A_X = (X_{max} - X_{min}) / 2$$
$$A_Y = (Y_{max} - Y_{min}) / 2$$

Using dynamic signal tracking to determinate amplitude and offset parameters will be more susceptible to noise compared to NLLSQR and DFT methods and may not provide best accuracy.

Therefore, if end-of-line parameters from NLLSQRT or DFT are available it is recommended to start with those values and smoothly adapt them with the dynamically calculated parameters by means of IIR filtering or moving average methods.

Once the amplitude and offset correction parameters are available, a first calibration can be applied as described in "How to Apply Calibration" section to obtain x_2 and y_2 values to calculate angle.

Phase Shift Parameter

To determine the phase shift parameter:

- Continuously calculate the approximate angle θ_{approx} with x_2 and y_2.

$$\theta_{approx} = \text{atan2}(y_2, x_2)$$

- Once θ_{approx} is equal to 45°, 135°, 225°, 315°, calculate and store the mean squared value M with the corresponding values of x_2 and y_2.

$$M_\theta = \sqrt{x_{2,\theta}^2 + y_{2,\theta}^2}$$
$$\text{for } \theta = 45, 135, 225, 315$$

Notice that for these angles $x_{2,\theta}^2 = y_{2,\theta}^2$, thus calculation of M can be simplified to:

$$M_\theta = \sqrt{2} \times |x_{2,\theta}|$$
$$\text{or}$$
$$M_\theta = \sqrt{2} \times |y_{2,\theta}|$$

- The phase shift can be calculated by using two set of values of M calculated at two angles spaced by 90°.

$$\Delta\varphi = 2 \times \text{atan2}(M_{135} - M_{45}, M_{135} + M_{45})$$
$$\Delta\varphi = 2 \times \text{atan2}(M_{135} - M_{225}, M_{135} + M_{225})$$
$$\Delta\varphi = 2 \times \text{atan2}(M_{315} - M_{225}, M_{315} + M_{225})$$
$$\Delta\varphi = 2 \times \text{atan2}(M_{315} - M_{45}, M_{315} + M_{45})$$

Full calibration can now be performed prior to angle calculation as explained in “How to Apply Calibration”.

CONCLUSION

An algorithm to optimally calculate angle with sine/cosine analog output signals from angle sensor was presented and different methods were proposed to find calibration parameters either in end-of-line or online in application. DFT or NLSQR methods can be used in end-of-line calibration to determine calibration parameters with high accuracy. To address lifetime and temperature drifts, it is recommended to use a combination of end-of-line calibration plus online dynamic compensation. As mentioned above the dynamic method based on maximum and minimum tracking is more subject to noise than end-of-line methods, thus filtering or averaging on calibration parameters is recommended.

Revision History

Number	Date	Description	Responsibility
–	May 10, 2021	Initial release	E. Casu
1	October 5, 2022	Updated Introduction and Calibration Parameters section	S. Pavlik